*Int J Heat Mass Transfer* Vol 33 No 11, pp 2511–2518, 1990
Printed in Great Britain

0017-9310/90 $3 00+0 00


# Simplified thermal models in laser and electron beam surface hardening

RENATO FESTA
ENEA CRE Casaccia TIB, IMA, S P Anguillarese, 301, 00060 Roma, Italy

and

ORONZIO MANCA and VINCENZO NASO
Dipartimento di Energetica, termofluidodinamica applicata e condizionamenti ambientali,
Università degli studi Federico II, P le Tecchio, 80125 Napoli, Italy

(*Received* 8 *June* 1989)

**Abstract**—Simplified thermal models in laser and electron beam surface hardening can be very useful to industrial users Hardening depth as a function of austenitization temperature and Peclet number is evaluated for one-dimensional stationary and two-dimensional uniform strip moving heat source problems without melting. Reliable correlations for predicting both hardening depths and maximum temperatures are given in terms of dimensionless meaningful parameters Calculated values are also compared with experimental results reported by other authors

## INTRODUCTION

Several solutions of the temperature field in a semi-infinite solid heated by a moving heat source can be applied in numerous mechanical processes, e.g in sliding solids and in surface treatments [1, 2] The latter have recently drawn a great deal of attention in industrial applications, due to the development of high-power laser surface treatments of metals. Therefore, it is felt a growing need of models predicting the resultant temperature field in the bodies, as simple as to be suitable for industrial users

A simplified model proposed by Chen and Lee [3] showed the existence of a critical velocity below which the effects of a Gaussian moving heat source become negligible and the temperature field is reasonably coincident with the one due to a stationary source Kou *et al* [4] developed a finite-difference numerical model for the three-dimensional heat flow in a semi-infinite body with a rectangular moving spot. Both surface heat losses and temperature dependence of surface absorptivity and thermal properties were taken into account Should thermal properties be assumed constant, a better agreement with the variable properties model is obtained if properties are evaluated at a temperature above the austenitization one rather than at room temperature. The calculated results based on high-temperature constant thermal properties were in a good agreement with experimental ones from the same authors By assuming no heat losses, constant absorptivity and properties, a very simple correlation predicting the maximum surface temperature for a square spot was given. It approaches that of the one-dimensional (1-D) model, for Peclet numbers higher than 10 La Rocca [5] gave a detailed analysis of the 1-D models for both a semi-infinite body and a finite slab Ashby and Easterling [6] and Li *et al.* [7] derived approximate solutions of the thermal field for a Gaussian moving heat source. Equations for the temperature field were combined with kinetic equations describing the homogenization of austenite and were able to predict the structure and the hardness of the treated region as a function of depth below the surface On the basis of experimental results, Davis *et al* [8] modified the solution for the hardening depth as a function of the power and velocity of a Gaussian beam proposed in ref. [9] Moreover, starting from the equation of heat conduction for a Gaussian moving source, they performed an asymptotic analysis involving several approximations, which yielded simplified solutions. Variable specific heat is accounted for by a quadratic approximation that cannot render the behaviour at the Curie point and makes the solutions difficult. For large Peclet numbers, the problem of the heat transfer around the contact region of sliding solids was reduced by Yuen [10] to a transient 1-D heat conduction in stationary bodies case. The simple model provided useful results only within the contact region. The same author [11] derived closed form expressions for the temperature fields which compared well with numerical solutions for Peclet numbers greater than 10. Festa *et al* [12] analysed the two-dimensional (2-D) solution for a uniform strip heat source moving at a constant velocity along the surface of a semi-infinite body and derived the ratio between the maximum temperature given by the 2-D model and that given by the 1-D model This ratio can be used for esti-

## NOMENCLATURE

| | |
|---|---|
| $b$ | hot spot half-width |
| $k$ | thermal conductivity |
| $K_0$ | modified Bessel function of the second kind, order zero |
| $n$ | polynomial degree |
| $Pe$ | Peclet number, equation (2b) |
| $q$ | heat flux |
| $t$ | time |
| $T$ | temperature |
| $T^+$ | dimensionless temperature, equation (2c) |
| $T^*$ | dimensionless temperature, equations (7) and (7′) |
| $v$ | heat source velocity |
| $\mathbf{v}$ | heat source vectorial velocity |
| $x, z$ | Cartesian coordinates |
| $X, Z$ | dimensionless Cartesian coordinates, equations (2a). |

Greek symbols

| | |
|---|---|
| $\alpha$ | thermal diffusivity |
| $\Delta$ | deviation, $100\,(Z_{h\,e}-Z_h)/Z_{h\,e}$ |
| $\zeta$ | variable, equations (2a) |
| $\mu$ | integration variable |
| $\xi$ | dimensionless time, equations (2a) |
| $\tau$ | dwell time. |

Subscripts

| | |
|---|---|
| 1-D,$\tau$ or 1 | stationary one-dimensional |
| 2-D,$v$ or 2 | moving two-dimensional |
| c | austenitization |
| e | experimental |
| h | hardening |
| m | maximum |

mating the range of Peclet number and depth within which the predictions of the two models are in a satisfactory agreement

The two-dimensional temperature field due to a uniform strip heat source over the surface of a semi-infinite body, $(2\text{-D})_v$, can be very useful in modelling both high power laser surface treatments with a square spot and electron beam treatments, where $xy$ beam scanning is carried out. As already noted, the simplest 1-D stationary model with uniform heat flux acting over a semi-infinite body for a finite time, $(1\text{-D})_\tau$, can be employed only for large Peclet numbers. The $(2\text{-D})_v$ solution, derived by Jaeger [1] and Blok [13], has been widely used (see for instance refs. [11, 12]).

In laser and electron beam solid state hardening various requirements must be fulfilled a limited power of the heat source, no surface melting and a given hardening depth must be taken into account. To this aim, simplified models are useful tools for industrial users They should predict hardening depths and maximum surface temperatures by easy and reliable correlations in terms of physically meaningful dimensionless parameters

In this paper reference is made to surface treatments of hardenable steels by solid state hardening and the following assumptions are made (i) no surface melting occurs, (ii) austenite transformation occurs at any depth at which the dynamic transformation temperature is attained for any length of time, (iii) cooling rate is fast enough to ensure hardening of all the austenitized zone. Using $(2\text{-D})_v$ and $(1\text{-D})_\tau$ models, hardening depth as a function of austenitization temperature and Peclet number is evaluated in ranges of the involved variables being of practical interest. For the $(1\text{-D})_\tau$ problem both maximum temperatures attained at various depths and hardening depths are non-dimensionally correlated with process parameters. For the $(2\text{-D})_v$ model a correlation is derived, which predicts hardening depths for given process parameters as well as the maximum surface temperature is simply correlated with the Peclet number Eventually, theoretical predictions are compared with experimental results reported in ref [4].

## MATHEMATICAL DESCRIPTION AND SOLUTION

The exact solutions of linear heat conduction problems in a semi-infinite isotropic homogeneous body at uniform initial temperature heated by a surface heat source are presented in ref. [5] for a stationary uniform constant heat flux over the whole surface during a finite time, $(1\text{-D})_\tau$, and in refs [12, 14] for a moving uniform constant strip $2b$ wide, $(2\text{-D})_v$ With reference to Figs. 1(a) and (b), respectively, thermal properties being assumed independent of position, the solution of the $(1\text{-D})_\tau$ problem is

$$T_{1\text{-D}\,\tau}(z,t) = \frac{2q_0\alpha^{1/2}}{k}\left\{t^{1/2}\,\mathrm{ierfc}\left[\frac{z}{(4\alpha t)^{1/2}}\right] - \delta(t)(t-\tau)^{1/2}\,\mathrm{ierfc}\left[\frac{z}{(4\alpha(t-\tau))^{1/2}}\right]\right\} \quad (1)$$

where

$$\delta(t) = \begin{cases} 0 & \text{for} \quad t \leqslant \tau \\ 1 & \text{for} \quad t > \tau \end{cases}$$

$\tau$ being the dwell time, that is the amount of time the spot on the surface is exposed to the uniform and constant heat flux (in the $(2\text{-D})_v$ problem it can be expressed as $\tau = 2b/v$) The solution of the $(2\text{-D})_v$ problem is

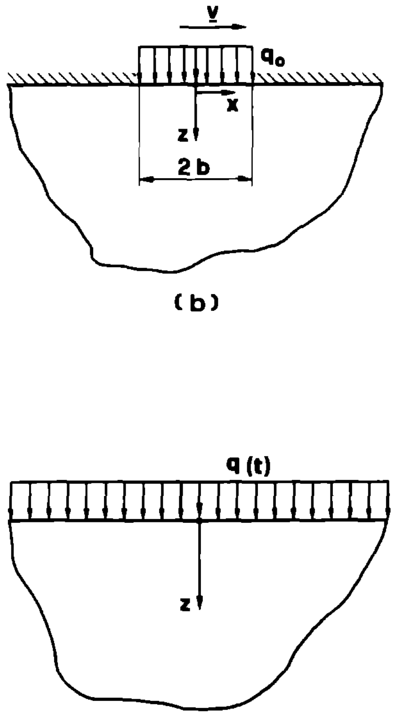


FIG. 1 Semi-infinite body heated over the surface: (a) uniform heat flux for a finite time amount, (b) moving heat source $2b$ wide

$$T_{2\text{-D},t}(x,z,t) = \frac{q_0\alpha^{1/2}}{2k\pi^{1/2}}\int_0^{\infty}\exp\left(-\frac{z^2}{4\alpha\mu}\right)\times\left\{\operatorname{erf}\left[\frac{v}{(2\alpha)^{1/2}}\frac{(x+b)/v-t+\mu}{(2\mu)^{1/2}}\right]-\operatorname{erf}\left[\frac{v}{(2\alpha)^{1/2}}\frac{(x-b)/v-t+\mu}{(2\mu)^{1/2}}\right]\right\}\frac{d\mu}{\mu^{1/2}}. \quad (1')$$

In equations (1) and (1′) $T$ is the temperature rise

The above equations can be written in dimensionless form Let

$$X = \frac{x}{(4\alpha\tau)^{1/2}},\quad Z = \frac{z}{(4\alpha\tau)^{1/2}},\quad \zeta = \frac{\mu}{\tau},\quad \xi = \frac{t}{\tau} \quad (2a)$$

$$Pe = \frac{vb}{2\alpha} = \left[\frac{2b}{(4\alpha\tau)^{1/2}}\right]^2 \quad (2b)$$

$$T^+ = \frac{T}{2bq_0/k} \quad (2c)$$

For the sake of brevity, $T^+_{1\text{-D},\tau}$ and $T^+_{2\text{-D},v}$ will be indicated as $T^+_1$ and $T^+_2$, henceforth

Then, equations (1) and (1′) can be written as

$$T^+_1(Z,\xi,Pe) = Pe^{-1/2}\{\xi^{1/2}\operatorname{ierfc}(Z/\xi^{1/2}) - \delta(\xi)(\xi-1)^{1/2}\operatorname{ierfc}[Z/(\xi-1)^{1/2}]\} \quad (3)$$

where

$$\delta(\xi) = \begin{cases}0 & \text{for } \xi \leqslant 1\\ 1 & \text{for } \xi > 1\end{cases}$$

and

$$T^+_2(X,Z,\xi,Pe) = \frac{1}{4(\pi Pe)^{1/2}}\int_0^{\infty}\exp\left(-\frac{Z^2}{\zeta}\right)\times\left\{\operatorname{erf}\left[Pe^{1/2}\frac{(X/Pe^{1/2}+1/2)-\xi+\zeta}{\zeta^{1/2}}\right]-\operatorname{erf}\left[Pe^{1/2}\frac{(X/Pe^{1/2}-1/2)-\xi+\zeta}{\zeta^{1/2}}\right]\right\}\frac{d\zeta}{\zeta^{1/2}} \quad (3')$$

Equations (3) and (3′) allow the evaluation of the depth, $Z_h$, where the austenitization temperature, $T^+_c$, is attained, provided $Z$ is considered as the dependent variable Thus, first of all the maximum temperature at every depth must be determined This can be accomplished by correlating $Z$ with the time, $\xi$, at which maximum temperature is attained For the $(1\text{-D})_\tau$ model La Rocca [5] gives

$$\xi(\xi-1)\ln[\xi/(\xi-1)] = 2Z^2. \quad (4)$$

For the $(2\text{-D})_v$ model, any $X$ undergoes the same temperature profiles as a function of $\xi$, for given $Z$ and $Pe$ It is suitable to write the equation for $X = Pe^{1/2}/2$, as shown in ref [12]

$$\exp(-2Pe)K_0\{2[(Z^2+Pe(1-\xi)^2)Pe]^{1/2}\} - K_0\{2[(Z^2+Pe\,\xi^2)Pe]^{1/2}\} = 0 \quad (4')$$

For the $(1\text{-D})_\tau$ solution, introducing $\xi = f(Z)$ from equation (4) into equation (3) yields

$$T^+_{1m}(Z,Pe) = F(Z)\,Pe^{-1/2} \quad (5)$$

For $Z = 0$, equation (5) becomes

$$T^+_{1m}(0,Pe) = (\pi\,Pe)^{-1/2} \quad (6)$$

Let

$$T^*_{1,m} = \frac{T^+_{1m}(Z,Pe)}{T^+_{1m}(0,Pe)} \quad (7)$$

dividing equation (5) by equation (6) yields

$$T^*_{1m}(Z) = \pi^{1/2}F(Z) \quad (8)$$

For the $(2\text{-D})_v$ solution, by introducing into equation (3′) $\xi = g(Z, Pe)$ from equation (4′), one gets

$$T^+_{2,m}(Z,Pe) = (\pi\,Pe)^{-1/2}G(Z,Pe). \quad (5')$$

Let

$$T^*_{2m} = \frac{T^+_{2,m}(Z,Pe)}{T^+_{1,m}(0,Pe)} \quad (7')$$

dividing equation (5′) by equation (6) yields

$$T^*_{2,m}(Z,Pe) = G(Z,Pe). \quad (8')$$

The dimensionless temperature in equations (7) and (7′) is physically meaningful and simpler than that in equation (2c); furthermore, it makes the maximum temperature independent of the Peclet number

The hardening depth can be obtained by inverting equations (8) and (8′) This yields, for the $(1\text{-D})_\tau$

$$Z_h = F^{-1}(T^*_{1m}) \qquad (9)$$

when $T^*_{1m}$ is equal to $T^*_c$ and, for the $(2\text{-D})_\iota$

$$Z_h = G_Z^{-1}(T^*_{2m}, Pe) \qquad (9')$$

when $T^*_{2,m}$ is equal to $T^*_c$

Since the inversion of equations (8) and (8′) cannot be performed analytically, it was carried out by empirical expressions. $Z_h$ values were calculated iteratively by equations (3) and (3′), as a function of $Pe$ and $T^+_{1,m}$ and $T^+_{2m}$, for $(1\text{-D})_\tau$ and $(2\text{-D})_v$, respectively

## RESULTS

The simplified models presented in the preceding section were used for a rather easy evaluation of hardening depth and maximum surface temperature in the high heat flux surface treatment of materials.

Hardening depth values were calculated at an approximation of $10^{-9}$ for the $(1\text{-D})_\tau$ and of $10^{-5}$ for the $(2\text{-D})_v$. The solution of equations (4) and (4′) was carried out by the Newton–Raphson method at an approximation of $10^{-9}$; the modified second kind Bessel functions, evaluated through the expressions given in Chap 9 of ref. [15], are affected by a relative error of magnitude $10^{-9}$ Maximum temperature values for the $(2\text{-D})_v$ were evaluated by a numerical integration of equation (3′) at an approximation of $10^{-5}$ The error function in equation (3′) and ierfc $(x)$ in equation (3) were evaluated by the expressions given in Chap 7 of ref. [15] Calculations were made for $10^{-2} \leqslant Pe \leqslant 10^2$, $0.05 \leqslant T^+_c \leqslant 1$ and $0 \leqslant Z \leqslant 4$

Results are first presented in diagram form and in terms of the dimensionless temperature defined in equation (2c), which allows a direct comparison between the predictions of $(1\text{-D})_\tau$ and $(2\text{-D})_v$ models Hardening depth as a function of Peclet number at various austenitization temperatures is presented in Figs 2 and 3 for $(2\text{-D})_v$ and $(1\text{-D})_\tau$ models, respectively They show that, for a given $Z_h$, the higher $T^*_c$ the lower $Pe$. This means that in the $(2\text{-D})_\iota$ the lower the heat flux the lower the velocity of the spot for a given width of the strip and a given thermal diffusivity. Figures show, also, that the lower $T^+_c$ the higher the slope of the curves. As expected, $Z_h$ values predicted by the two models are in good agreement for high Peclet numbers (> 10) while at low values (< 1) the $(1\text{-D})_\tau$ overpredicts the hardening depth. Figure 2 suggests that significant hardening depths can be achieved with a practically stationary source, that is for very low Peclet numbers.

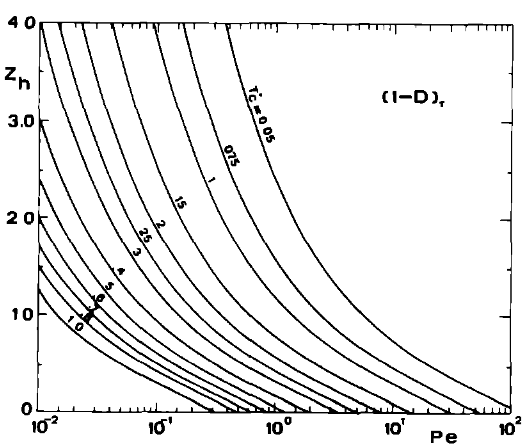

FIG 2 Hardening depth vs Peclet number at various austenitization temperatures in (2-D), model

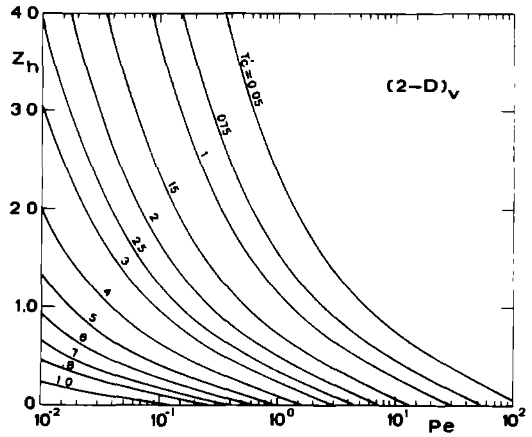

FIG 3 Hardening depth vs Peclet number at various austenitization temperatures in $(1\text{-D})_\tau$ model

Maximum surface temperatures vs Peclet numbers, according to the $(2\text{-D})_\iota$ model, are presented in Fig 4, which allows an easy comparison between the maximum temperature attained in the material and its melting temperature.

Hardening depth vs $T^+_c$ at various $Pe$ is plotted in Fig. 5, which suggests considerations similar to those resulting from Fig 2. Furthermore, for $10 \leqslant Pe$, the high slope of the curves discloses how the accurate prediction of hardening depth strongly depends on the good knowledge of austenitization temperatures

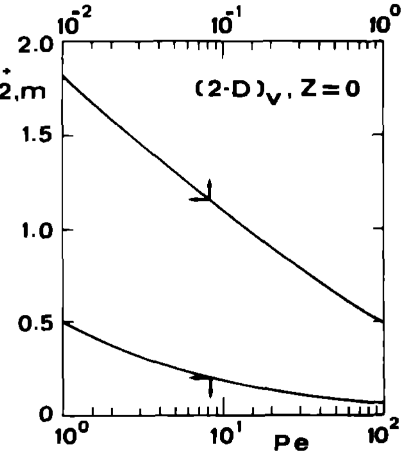

FIG 4 Maximum surface temperature vs Peclet number in (2-D), model

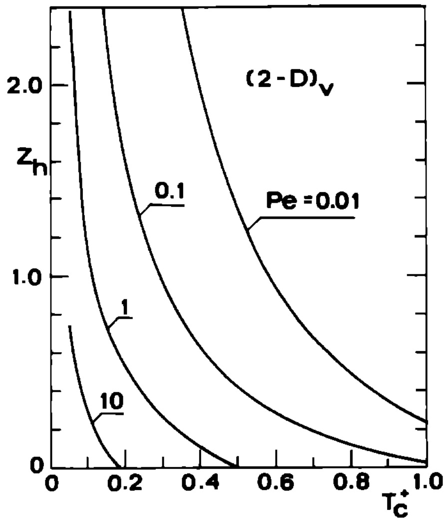


FIG 5 Hardening depth vs austenitization temperature at various Peclet numbers in (2-D)r model

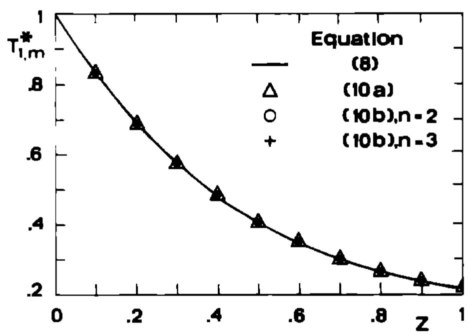


FIG 6 Maximum temperature vs depth in (1-D)r model.

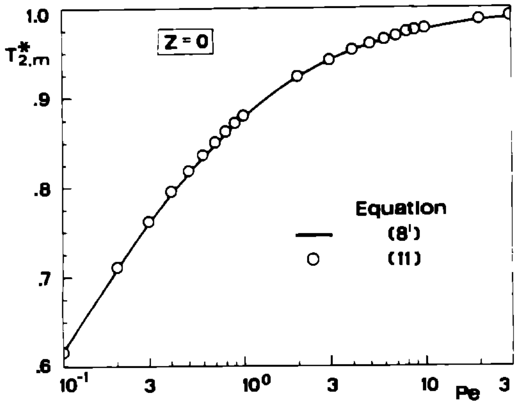


FIG. 7. Maximum surface temperature vs Peclet number in (2-D)r model

*Maximum temperature correlations*

In the following, correlations for the prediction of maximum temperature are presented. They are based on both models and are within ranges of variables of practical interest. For the (1-D)r problem the maximum temperature attained at various depths has been correlated to the depth by the following equations.

$$T^*_{1,m} = a_0 + a_1 \exp(a_2 Z) \quad (10a)$$

for $0 \leqslant Z \leqslant 1$ and

$$T^*_{1,m} = \exp[-(a_0 + a_1 Z + a_2 Z^2 + \cdots + a_n Z^n)] \quad (10b)$$

for $0 \leqslant Z \leqslant 1$ and $n = 2$ and 3, the relevant coefficients being reported in Table 1. This discloses high values of regression coefficients for all the proposed correlations It can be noted that equation (10a) is suitably invertible for an easy evaluation of $F^{-1}(T)$ In Fig. 6 the maximum temperature $T^*_{1,m}$ is plotted vs depth, $Z$, according to equation (8), together with some values derived from equations (10). The figure clearly shows that in solid state hardening of steels $Z_h$ values cannot be higher than about 0.4 since this value yields $T^*_{1,m} \simeq 0.5$, namely at the surface the melting temperature ($\simeq 1600°C$) is attained for an austenitization temperature of 800°C.

The maximum temperature attained at the surface was correlated with Peclet number in the form

$$T^*_{2,m} = [0.14304 + 0.99566(Pe + 0.0205)^{-0.7}]^{-1} \quad (11)$$

for $0.1 \leqslant Pe \leqslant 30$, with $r^2 = 0.99992$ and s.e $= 8.8 \times 10^{-4}$ In Fig. 7 the maximum temperature $T^*_{2,m}$ is plotted vs Peclet number at $Z = 0$, according to equation (8′), together with the same values derived from equation (11).

*Hardening depth correlations*

The hardening depth was correlated with the austenitization temperature. For the (1-D)r the expression is

$$Z_h = a_0 + a_1 \ln T^*_c + \cdots + a_n (\ln T^*_c)^n \quad (12)$$

for $0.2 \leqslant T^*_c \leqslant 1$, where $T^*_c = T^*_{1,m}$. Relevant coefficients are reported in Table 2

For the (2-D)r problem the correlation is

$$Z_h = a_0 + a_1 \ln T^*_c + \cdots + a_n (\ln T^*_c)^n \quad (12')$$

for $0.1 \leqslant T^*_c \leqslant 1$, where $T^*_c = T^*_{2,m}$. The coefficients

Table 1 Parameter and coefficients of equations (10a) and (10b)

| Equation | $n$ | $a_3$ | $a_2$ | $a_1$ | $a_0$ | $r^2$ | s.e |
|---|---|---|---|---|---|---|---|
| (10b) | 3 | −0 27842 | −0 10529 | 1 93640 | −0 00831 | 0 99994 | $4.4 \times 10^{-3}$ |
| (10b) | 2 | — | −0 53316 | 2 10568 | −0.02121 | 0 99977 | $8.3 \times 10^{-3}$ |
| (10a) | — | — | −2 24530 | 0.89516 | 0.11563 | 0 99973 | $3.4 \times 10^{-3}$ |

Table 2 Parameter and coefficients of equation (12)

| $n$ | $a_3$ | $a_2$ | $a_1$ | $a_0$ | $r^2$ | s.e |
|---|---|---|---|---|---|---|
| 3 | −0 12348 | −0.15495 | −0 59076 | −0 0020535 | 0 99999 | $8\ 6\times10^{-4}$ |
| 2 | — | 0 14465 | −0.39982 | 0 021748 | 0 99890 | $1\ 0\times10^{-2}$ |
| 1 | — | — | −0 62864 | −0 036393 | 0.98954 | $3\ 1\times10^{-2}$ |

in equation (12′) depend on Peclet number. This required a preliminary evaluation of the above-mentioned coefficients through the same equation (12′) in a suitable range of $Pe$. Values for $n = 2$ and 3 are presented in Figs 8 and 9, respectively Figures show that, for $Pe \to \infty$, the values of coefficients in equation (12′) tend to those of the corresponding coefficients in equation (12) for the $(1\text{-D})_\tau$ problem This agrees well with results presented in refs [11, 12]. Coefficients were correlated with Peclet numbers, in the range $0\ 1 \leqslant Pe \leqslant 30$, by the following expressions.

For $n = 2$.

$$a_0 = 0\ 027187-0\ 071849Pe^{-0\ 58}+0.019715(Pe^{-0\ 58})^2 \quad (13)$$

with a regression coefficient ($r^2$) of 0 99866 and a standard error (s.e.) of $8.6\times10^{-4}$;

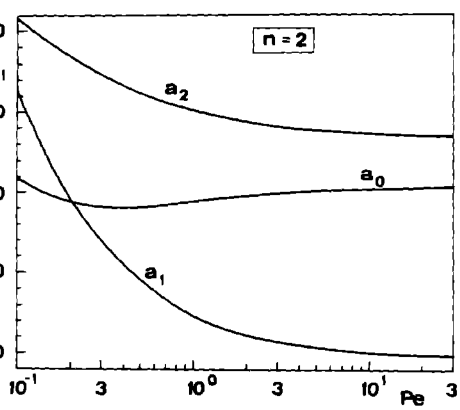


Fig 8 Coefficients in equation (12′) for $n = 2$

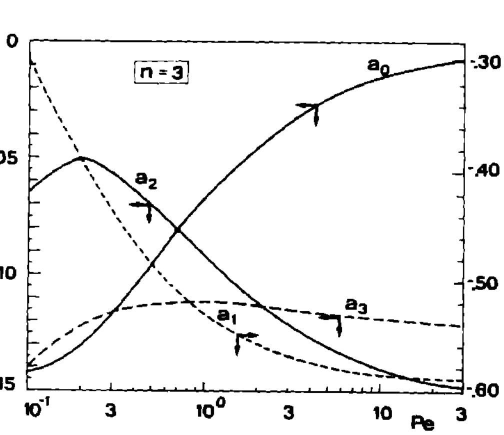


Fig 9 Coefficients in equation (12′) for $n = 3$

$$a_1 = 0.40984+0\ 10780(Pe+0.05)^{-0\ 95} \quad (14)$$

with $r^2 = 0\ 99986$ and s e $= 1\ 9\times10^{-3}$;

$$a_2 = 0.12969+0\ 078220(Pe+0\ 025)^{-0\ 65} \quad (15)$$

with $r^2 = 0\ 99946$ and s e. $= 1\ 7\times10^{-3}$

For $n = 3$

$$a_0 = [46.98815-30\ 71564(Pe+3)^{0\ 5}]^{-1} \quad (16)$$

with $r^2 = 0.99997$ and s.e $= 2.0\times10^{-1}$,

$$a_1 = -0.59217+0.070263(Pe+0.14)^{-1} \quad (17)$$

with $r^2 = 0\ 99981$ and s e. $= 1.1\times10^{-3}$,

$$a_2 = -0.16391+0\ 089267Pe^{-0\ 55}-0.017464(Pe^{-0\ 55})^2 \quad (18)$$

with $r^2 = 0.99967$ and s.e $= 7.2\times10^{-4}$,

$$a_3 = -0\ 12706+0\ 029177Pe^{-0\ 475}-0\ 016133(Pe^{-0\ 475})^2+0\ 016354(Pe^{-0\ 475})^3 \quad (19)$$

with $r^2 = 0\ 99948$ and s.e. $= 1.5\times10^{-4}$

Most coefficients correlations are of the first degree, except equations (13), (18) and (19). In the following, correlations of lower degree are proposed. Instead of equation (13), coefficient $a_0$ can be correlated, in the range $0.5 \leqslant Pe \leqslant 30$, by

$$a_0 = 0.019117-0\ 10191(Pe+1.3)^{-1} \quad (20)$$

with $r^2 = 0\ 99986$ and s.e $= 2.4\times10^{-4}$. Coefficient $a_2$, with $n = 3$, can be correlated, in the range $0\ 2 \leqslant Pe \leqslant 30$, by

$$a_2 = -0\ 15206+0\ 12272(Pe+1)^{-1} \quad (21)$$

with $r^2 = 0.99962$ and s e. $= 6.7\times10^{-4}$. Similarly, equation (19) can be reduced to

$$a_3 = -0.13001+0.033062Pe^{-0\ 38}-0.015596(Pe^{-0\ 38})^2 \quad (22)$$

with $r^2 = 0.99835$ and s.e. $= 2.5\times10^{-4}$, in the same range of Peclet number

*Comparison with experimental results*

In Table 3 experimental values of hardening depth obtained with a square laser beam by Kou *et al* [4] are compared with those calculated by equations (8), (8′), (12) and (12′) of this paper.

Experiments were carried out with $k = 29.5$ W m$^{-1}$ K$^{-1}$, $\alpha = 4.1\times10^{-6}$ m$^2$ s$^{-1}$, $T_c = 780$ K; $b = 12$ mm; $v = 38$ mm s$^{-1}$, $q_0 = 4\ 00\times10^7$, $3.40\times10^7$,

Table 3. Comparison of numerical results with experimental data from ref. [4]

| $T_c^*$ | $Z_{h,c}$ from ref. [4] | (1-D)$_\tau$ equation (8) $Z_h$ | $\Delta$ | (2-D)$_r$ equation (8′) $Z_h$ | $\Delta$ | (1-D)$_\tau$ equation (12), $n = 1$ $Z_h$ | $\Delta$ | (1-D)$_\tau$ equation (12), $n = 2$ $Z_h$ | $\Delta$ | (2-D)$_r$ equation (12′), $n = 2$ equations (13)–(15) $Z_h$ | $\Delta$ | (2-D)$_r$ equation (12′), $n = 3$ equations (16)–(19) $Z_h$ | $\Delta$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0.533 | 0.355 | 0.340 | −4.3 | 0.335 | −5.8 | 0.359 | 1.1 | 0.340 | −4.5 | 0.328 | −7.6 | 0.335 | −5.7 |
| 0.627 | 0.241 | 0.252 | 4.6 | 0.247 | 2.4 | 0.257 | 6.5 | 0.252 | 4.7 | 0.237 | −1.6 | 0.248 | 2.7 |
| 0.702 | 0.177 | 0.192 | 8.5 | 0.186 | 5.2 | 0.186 | 5.1 | 0.192 | 9.1 | 0.179 | 1.0 | 0.188 | 6.3 |
| 0.775 | 0.131 | 0.139 | 6.0 | 0.133 | 1.3 | 0.124 | −5.7 | 0.140 | 7.1 | 0.130 | −0.7 | 0.135 | 3.2 |

$3.04\times10^7$, $2.76\times10^7$ W m$^{-2}$, the resulting Peclet number being equal to 27.9. Thermal properties were assumed at 800°C, according to the suggestions of Kou *et al.* [4]. The agreement between data from different sources is generally good. The table shows that experimental data agree with numerical values obtained from equation (8′) better than with those from equation (8), in spite of the large Peclet numbers. Furthermore, for the (1-D)$_\tau$ problem, on average the deviations corresponding to the first degree polynomial correlation ($n = 1$) in equation (12) are smaller than those corresponding to $n = 3$. Finally, for the (2-D)$_r$ model, predictions by the second degree polynomial correlation from equation (12′) agree with experimental data better than those by the third degree one for $T_c^*$ higher than 0.62. On the contrary, values predicted by the third degree correlation seem to be in better agreement for lower values of $T_c^*$.

## CONCLUSIONS

Two models for the prediction of hardening depths and maximum temperatures at various depths in laser and electron beam surface hardening have been analysed. Reference was made to uniform and constant heat flux acting for a finite time over a semi-infinite solid, (1-D)$_\tau$, and to a uniform strip heat source moving at a constant velocity over a semi-infinite body, (2-D)$_r$.

Diagrams plotting hardening depth as a function of austenitization temperature and Peclet number show that, for $Pe > 10$, $Z_h$ can be predicted by the simpler (1-D)$_\tau$ model if a negligible error is accepted. For $Pe < 10$, errors associated with the (1-D)$_\tau$ predictions become higher and the (2-D)$_r$ model is to be resorted to get more useful results.

All results are valid if no surface melting occurs. To this aim both a diagram and a correlation for the maximum surface temperature vs Peclet number, in the range $0.1 \leqslant Pe \leqslant 30$, are provided for the (2-D)$_r$ model.

For the (1-D)$_\tau$ model the maximum dimensionless temperature attained at any depth depends only on the dimensionless depth. It is plotted and simply correlated to the depth to show that in laser and electron beam solid state hardening of steels hardening depth cannot be higher than about 0.4, otherwise surface melting occurs.

On the grounds of the afore-mentioned results, the correlation for predicting the hardening depth as a function of the dimensionless meaningful parameters Peclet number and austenitization temperature are given in ranges of variables of practical interest ($0.1 \leqslant Pe \leqslant 30$, $0.15 \leqslant T_c^* \leqslant 1.0$, $0 \leqslant Z_h \leqslant 1.0$).

Results from diagrams and correlations were compared with experimental data available from ref. [4]. Agreement is generally good, the maximum deviation being 9.1%.

## REFERENCES

1. J. C. Jaeger, Moving sources of heat and temperature at sliding contacts, *Proc. R. Soc. New South Wales* **76**, 203–224 (1942).

2 D Rosenthal, The theory of moving sources of heat and its application to metal treatments, *Trans Am Soc Mech Engrs* **68**, 849–866 (1946)
3 I Chen and S Lee, Transient temperature profiles in solids heated with scanning laser, *J Appl Phys* **54**, 1062–1066 (1983)
4 S Kou, D K Sun and Y P Le, A fundamental study of laser transformation hardening, *Metall Trans A* **14A**, 643–653 (1983)
5 A V La Rocca, Models of thermal fields with high entering flux for laser processing, presented at 5th Int Symp on Gas Flow and Chem Lasers, Oxford (1984)
6 M F Ashby and K E. Easterling, The transformation hardening of steel surfaces by laser beams—I Hypo-eutectoid steels, *Acta Metall.* **32**, 1935–1948 (1984)
7 W B Li, K E Easterling and M F Ashby, Laser transformation hardening of steel—II. Hypereutectoid steels, *Acta Metall* **34**, 1533–1543 (1986)
8 M Davis, P. Kapadia, J Dowden, W M Steen and C H G Courtney, Heat hardening of metals surfaces with a scanning laser beam, *J. Phys D Appl Phys* **19**, 1981–1997 (1986)
9 W M Steen and C H G Courtney, Surface heat transfer of $En_8$ steel using a 2 kW continuous-wave $CO_2$ laser *Met Technol* **6**, 456–462 (1979)
10 W Y D Yuen, On the heat transfer of a moving composite strip compressed by two rotating cylinders, *J Heat Transfer* **107**, 541–548 (1985)
11 W Y D Yuen, Heat conduction in sliding solids, *Int J Heat Mass Transfer* **31**, 637–646 (1988)
12 R Festa, O Manca and V Naso, A comparison between models of thermal fields in laser and electron beam surface processing, *Int J Heat Mass Transfer* **31**, 99-106 (1988)
13 H Blok, Theoretical study of temperature rise at surfaces of actual contact under oiliness conditions, *Inst Mech Engrs Gen Disc Lubrication* **2**, 222–235 (1937)
14 H S Carslaw and J C Jaeger, *Conduction of Heat in Solids*, 2nd Edn, p 269 Oxford University Press Oxford (1959)
15 M Abramowitz and I. A Stegun, *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*, Appl Math Ser., Vol 55 National Bureau of Standards, Washington, DC (1964)

## MODELES THERMIQUES SIMPLIFIES POUR LE DURCISSEMENT SUPERFICIEL PAR LASER ET FAISCEAUX D'ELECTRONS

**Résumé**—Les modéles thermiques simplifiés pour les traitements superficiels de durcissement par laser et faisceaux d'électrons peuvent être très utiles pour les utilisateurs industriels La profondeur de la zone durcie en fonction de la température d'austénitisation et du nombre de Peclet a été évaluée L'analyse, limitée au cas de non fusion superficielle, a été basée sur les solutions analytiques des problèmes des sources de chaleur monodimensionnelles stationnaires et bi-dimensionnelles d'une bande uniforme en mouvement Des corrélations pour la prédiction soit de la profondeur de durcissement soit de la température maximale ont été données au moyen de paramètres adimensionnels physiquement significatifs Les valeurs calculées ont aussi été comparées avec des résultats reportés par d'autres auteurs

## VEREINFACTE TERMISCHE MODELLE FUR DIE OBERFLACHENAUSHARTUNG MITTELS LASER- UND ELEKTRONENSTRAHLEN

**Zusammenfassung**—Vereinfachte termische Modelle fur Oberflächenaushartungsbehandlungen mittels Laser- und Elektronenstrahlen haben sich sehr nutzlich erwiesen Die Tiefe der ausgeharteten Zone ist in Abhangikeit von der Austenitisierungstemperatur und Peclet's Zahl berechnet worden Die Analyse wird auf die Falle ohne Oberflächenschmelzen beschrankt Die angewandten Modelle stützen sich auf die analytischen Lösungen im Fall einer stationaren monodimensionallen Warmequellen und im Fall eines bidimensionällen gleichformigen Bandes das im Bewegung sich befindet Die Beziehungen, die Vorausage sowohl der Aushartungstiefe als auch der hochsten erreichten Temperatur erlauben, sind mittels adimensionallen von physicalischer Bedeutung Parameter angegeben worden Die berechneten Werten sind also mit den Ergebnissen anderer Authoren vergliechen worden

## УПРОЩЕННЫЕ ТЕПЛОВЫЕ МОДЕЛИ ПОВЕРХНОСТНОГО УПРОЧНЕНИЯ С ПОМОЩЬЮ ЛАЗЕРНЫХ И ЭЛЕКТРОННЫХ ПУЧКОВ

**Аннотация**—Упрощенные тепловые модели поверхностного упрочнения с помощью лазерных и электронных пучков могут успешно применяться в промышленности. Оценивается глубина упрочнения как функция температуры аустенизации и числа Пекле в задачах одноменого стационарного и двумерного однородного движения пластинчатых источников тепла без учета плавления Приводятся надежные соотношения для расчета глубин упрочнения и максимальных температур через безразмерные параметры. Расчетные значения сравниваются с экспериментальными результатами, полученными другими авторами